FLUID FLOW

Mechanical Energy Balance

$$g\Delta Z + \int \frac{dp}{\rho} + \Delta\left(\frac{V^2}{2}\right) = W_o - \sum F$$

- potential energy change
- expansion work
- kinetic energy change
- work added/ subtracted by pumps or compressors
- sum of friction losses

Note that the balance is per unit mass. In differential form:

$$dp = -\rho\left(g \cdot dZ - V \cdot dV - \delta F + \delta W_o\right)$$

FLUID FLOW

Mechanical Energy Balance

Divide by *dL*, (L is the length of the pipe)

$$\left.\frac{dp}{dL}\right|_{Tot} = -\rho g \frac{dZ}{dL} + \rho V \frac{dV}{dL} + \rho \frac{\delta F}{\delta L} - \rho \frac{\delta W_o}{\delta L}$$

or:

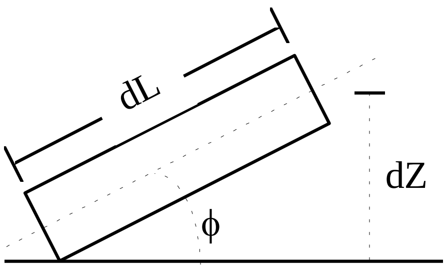
$$\left.\frac{dp}{dL}\right)_{Tot} = \left.\frac{dp}{dL}\right)_{elev} + \left.\frac{dp}{dL}\right)_{accel} + \left.\frac{dp}{dL}\right)_{frict}$$

$\frac{\delta W_o}{\delta L}$ is usually ignored, as the equation applies to a pipe section.

The above equation is an alternative way of writing the mechanical energy balance. It is not a different equation

Mechanical Energy Balance

Potential energy change:



$$g\frac{dZ}{dL} = g\sin\phi$$

Friction Losses:

Fanning equation:

$$dF = \frac{2V^2 f}{D}dL$$

This equation applies to single phase fluids.

The friction factor is obtained from the "Moody Diagram" (see PT page 487).

Mechanical Energy Balance

Friction factor equations. (Useful for computers and Excel)

$$f = \frac{16}{\mathrm{Re}}$$

Laminar Flow

$$f = \frac{0.046}{\mathrm{Re}^{a}}$$

Smooth pipes: a = 0.2
Iron or steel pipes: a = 0.16

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{\varepsilon}{3.7D} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right)$$

Colebrook equation for turbulent flow.

Equivalent length of valves and fittings.

Pressure drop for valves and fittings is accounted for as equivalent length of pipe.
See PT&W for a table containing these values (page 490).

Mechanical Energy Balance - Fluid Flow

Scenario I

Need pressure drop in known pipes (pump or compressor is not present.)

Incompressible Flow

a) Isothermal (ρ is constant)

$$\left.\frac{dp}{dL}\right|_{Tot} = -\rho\left(g\cdot\frac{dZ}{dL} + V\cdot\frac{dV}{dL} + \frac{\delta F}{\delta L}\right)$$

for a fixed ρ $\Rightarrow$ V constant $\Rightarrow$ $dV = 0$

Integral form:

$$\Delta p = -\rho\left[g\cdot\Delta Z + 2V^2\cdot f\cdot\frac{L+L_e}{D} + \sum F\right]$$

b) Nonisothermal

It will not have a big error if you use $\rho(T_{average})$

Mechanical Energy Balance - Fluid Flow

Compressible Flow

a) Relatively small change in T (known)

For small pressure drop (something you can check after you are done) can use Bernoulli and fanning equation as follows

$$g \cdot dz + v \cdot dp + d\left(\frac{V^2}{2}\right) = -\delta F$$

$$\frac{g}{v^2} \cdot dz + \frac{1}{v} \cdot dp + \frac{V}{v^2} \cdot dV = -\frac{\delta F}{v^2}$$

V = Velocity

G = Mass flow (Kg/hr)

v = Specific volume (m^3/Kg)=1/ρ

A = Cross sectional area

Note: $V = v \cdot \frac{G}{A}$

Mechanical Energy Balance - Fluid Flow

Compressible Flow. Relatively small change in T (known)

$$\frac{g}{v^2}\cdot dz+\frac{1}{v}\cdot dp+\left(\frac{G}{A}\right)\cdot\frac{dV}{v}=-\frac{\delta F}{v^2}=-2\cdot f\cdot\left(\frac{G}{A}\right)^2\frac{dL}{D}$$

Now put in integral form

$$g\int\frac{dz}{v^2}+\int\frac{dp}{v}+\left(\frac{G}{A}\right)^2\int\frac{dV}{V}=-2\cdot\left(\frac{G}{A}\right)^2\cdot\frac{1}{D}\cdot\int f\,dL$$

Assume:

$$T_{av}=\frac{T_{in}+T_{out}}{2}\qquad f_{av}=\frac{f_{in}+f_{out}}{2}$$

$$\rho_{av}=\frac{\rho(T_{in},P_{in})+\rho(T_{out},P_{out})}{2}\qquad f_{av}=\frac{f(T_{in},P_{in})+f(T_{out},P_{out})}{2}$$

Mechanical Energy Balance - Fluid Flow

Compressible Flow. Relatively small change in T (known)

The integral form will be:

$$\rho_{av}^2 g\Delta z + \int_{in}^{out} \frac{dp}{v} + \left(\frac{G}{A}\right)^2 \ln\left(\frac{V_{out}}{V_{in}}\right) = -2\left(\frac{G}{A}\right)^2 f_{av} \frac{L}{D}$$

Recall: $p\,v = \dfrac{Z\,R\,T}{M}$ M: Molecular weight

Then: $v \cong Z_{av} \dfrac{RT_{av}}{pM}$

and

$$\int \frac{dp}{v} = \frac{M}{Z_{av} RT_{av}} \int p \cdot dp = \frac{M}{2 \cdot Z_{av} RT_{av}} \left(p_{out}^2 - p_{in}^2\right)$$

Mechanical Energy Balance - Fluid Flow

Compressible Flow. Relatively small change in T (known)

Substitute in the integral form:

$$\rho_{av}^2 g\cdot\Delta z+\frac{M}{2\cdot Z_{av}RT_{av}}\left(p_{out}^2-p_{in}^2\right)+\left(\frac{G}{A}\right)^2\ln\left(\frac{V_{out}}{V_{in}}\right)=-2\left(\frac{G}{A}\right)^2 f_{av}\frac{L}{D}$$

Since: $$\frac{V_{out}}{V_{in}}=\left(\frac{Z_{out}\cdot T_{out}}{Z_{in}\cdot T_{in}}\right)\cdot\frac{p_{in}}{p_{out}}$$

we get

$$p_{out}=\left[p_{in}^2-2\frac{Z_{av}RT_{av}}{M}\left\{2\left(\frac{G}{A}\right)^2\cdot f_{av}\cdot\frac{L}{D}+\left(\frac{G}{A}\right)^2\ln\left(\frac{Z_{out}T_{out}p_{in}}{Z_{in}T_{in}p_{out}}\right)+\rho_{av}^2\cdot g\cdot\Delta z\right\}\right]^{\frac{1}{2}}$$

Mechanical Energy Balance - Fluid Flow

Compressible Flow. Relatively small change in T (known)

This is an equation of the form: $p_{out} = F(p_{out})$

Algorithm:

a) Assume $p_{out}^{(1)}$

b) Use formula to get a new value $p_{out}^{(2)} = F\left(p_{out}^{(1)}\right)$

c) Continue using $p_{out}^{(i+1)} = F\left(p_{out}^{(i)}\right)$

until $$\frac{p_{out}^{(i+1)} - p_{out}^{(i)}}{p_{out}^{(i)}} \leq \varepsilon$$

OR BETTER: Use Solver in EXCEL, or even use PRO II, or any other fluid flow simulator.

Mechanical Energy Balance - Fluid Flow

Compressible Flow. Relatively small change in T (known)

The above algorithm can be applied for cases where

$$\frac{p_{out} - p_{in}}{p_{in}} \leq 0.2 - 0.3$$

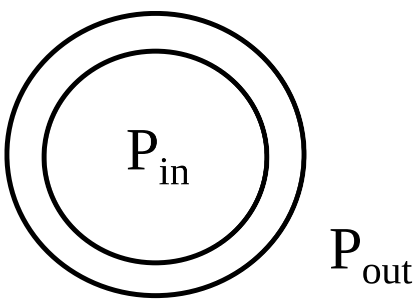
For longer pipes, break the pipe into smaller sections

PIPING STRENGTH

Bursting pressure of a pipe

$$P_b = 2S_T \frac{t_m}{D_m}$$

D_m= Mean Diameter
t_m= Wall Thickness
S_t= Tensile Strength (properties of material and fabricate)
P_b= Bursting pressure



$$P_b = P_{in} - P_{out}$$

PIPING STRENGTH

Safe Working Pressure

$$P_S = 2S_S \frac{t_m}{D_m}$$

We substitute with a safe working stress, $S_s < S_T$

Range of S_s = 6500-9000 psi (T < 250 °F)

(Low end) butt-welded → 6500

lap-welded (High end) → 9000

Schedule of a Pipe (American Standard Association)

There are 10 Sch numbers:

10, 20, 30, **40**, 60, 80, 100, 120, 140, 160

PIPING STRENGTH

Schedule of a Pipe (American Standard Association)

You specify a pipe by giving the diameter and the Schedule

- Get pressure inside , P_{in} (psia)
- $P_S = P_{in} - 14.696$
- $\alpha = 1000 \frac{P_S}{S_S}$; S_S ==> Characteristic of pipe (6500 – 9000 psi)

- Pick lower possible Sch standard.

$$Sch > a$$